Extraordinary Baryon Fluctuations and the QCD Tricritical Point

Sean Gavin
Department of Physics and Astronomy, Wayne State University, Detroit, MI, 48202
(February 9, 2008)

The dynamic separation into phases of high and low baryon density in a heavy ion collision can enhance fluctuations of the net rapidity density of baryons compared to model expectations. We show how these fluctuations arise and how they can survive through freezeout.

QCD can exhibit a first order phase transition at high temperature and baryon density, culminating in a tricritical point [1,2]. Specifically, below the tricritical point, a phase coexistence region separates distinct phases of QCD matter at different baryon densities, as shown in fig. 1. Stephanov, Rajagopal and Shuryak have pointed out that critical fluctuations of E_T and similar meson measurements can lead to striking signals at the tricritical point in relativistic heavy ion collisions [3].

We suggest that measurements of fluctuations of the net baryon number in nuclear collisions can help establish the first order coexistence region and, ultimately, the tricritical point. We characterize baryon fluctuations by the variance $\mathcal{V}_B \equiv \langle N_B^2\rangle - \langle N_B\rangle^2$, where $N_B = N - \overline{N}$ is the net baryon number in one unit of rapidity, obtained from the baryon N and antibaryon $\overline{N}$ distributions; the average is over events. Ordinarily, net baryon fluctuations in thermal and participant nucleon models of heavy ion collisions satisfy

$$\mathcal{V}_B^0 \approx TV\partial\rho_B/\partial\mu_B \approx N + \overline{N}, \tag{1}$$

where the second equality holds for an ideal gas [4,5]. In contrast, we argue below that enhanced fluctuations occur if the expansion of the system quenches the matter from an initial high density state into the phase coexistence region. At the tricritical point, these fluctuations *diverge* because $V\partial\rho_B/\partial\mu_B = -\partial^2\Omega/\partial\mu_B^2 \rightarrow \infty$ in (1), where Ω is the free energy.

Baryon fluctuation measurements add new leverage to a search for the first order region, complimenting information from pion and kaon interferometry, intermittency and wavelet analyses [6]. The latter measurements probe the spatial structure introduced by phase separation and droplet formation. By comparison, baryon fluctuations are weakly dependent on the morphology of the mixed phase, because they do not rely on distinct droplets escaping a rather dense system.

In this paper we explore the onset of baryon fluctuations and the possibility of their dissipation in nuclear collisions. That an order parameter such as the baryon density should undergo extraordinary fluctuations during a phase transition comes as no surprise – critical opalescence results from an analogous divergence of density fluctuations. Furthermore, less extreme but nevertheless measurable density fluctuations are familiar in condensed matter systems that are rapidly quenched into a phase coexistence region [7]. Perhaps more surprising is the possibility that observable baryon fluctuations can survive the subsequent evolution of the system.

As motivation, we start by describing how phase separation can produce observable fluctuations. We then use a spinodal decomposition model [7] to illustrate how a highly supercooled system can produce large fluctuations. Finally, we ask whether diffusion can dissipate these fluctuations before they are detected. Generally, there are two ways the dynamics can obscure large net baryon fluctuations in a subregion of the system. First, particles can diffuse throughout the fluid, diluting any "hot spots" and their consequent fluctuations. Second, fluid flow can carry the particles away to similar effect. We focus on the effect of diffusion because transport theory estimates indicate that the relevant diffusion coefficient can be large [8,9]. In principle, chemical reactions introduce dissipation by annihilating and creating baryon-antibaryon pairs. However, these reactions cannot affect the net baryon number, which is conserved, although they do affect the individual baryon and antibaryon fluctuations [5].

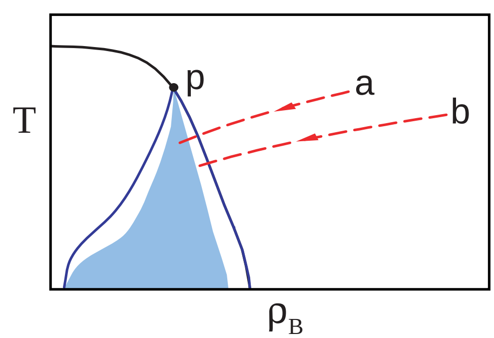

FIG. 1. Schematic QCD phase diagram [1,2] including the tricritical point (p) and the spinodal region (shaded). The dashed curves indicate trajectories followed by ion collisions.

Figure 1 shows the expected QCD phase diagram at high temperature T and baryon density ρ_B. A phase coexistence region lies below the tricritical point. Inside this region, uniform matter must eventually break up into distinct domains containing the high and low density phases respectively. Within the coexistence region is the spinodal region (the shaded area). There, uniform matter is mechanically unstable, so that small fluctuations

can rapidly generate bubbles throughout the fluid. Matter is metastable between the spinodal and coexistence boundaries, so that fluctuations must overcome an energy barrier to nucleate bubbles. Phase separation by spinodal decomposition and bubble nucleation [7] have been discussed in the context of heavy ion collisions [10,11], albeit with different phase diagrams in mind.

Ideally, a heavy ion collision will produce extraordinary fluctuations if experimenters can adjust the beam energy and the ion combination to produce initial baryon densities and temperatures within the spinodal region or, optimally, at the tricritical point. Alternatively, fluctuations can arise if the expansion of the heavy ion system rapidly quenches a high density system deeply into the spinodal region. The dashed curves in fig. 1 show one such quenching trajectory (a) and one trajectory that only reaches the nucleation region (b). Either process may produce phase separation far from equilibrium [11].

In spinodal decomposition, runaway density fluctuations rapidly contort and compress the high density fluid into filamentary regions. These runaway modes appear because the fluid is dynamically unstable, with $\partial\rho_B/\partial\mu_B < 0$ inside the spinodal region [7]. These modes grow exponentially until the density outside the regions reaches the equilibrium density for that temperature, ρ_h, at the low-density boundary of the phase coexistence curve. After this burst of nonequilibrium growth, phase separation in an ion collision can proceed smoothly as the system expands and rarefies. Nucleation may proceed more uniformly, with perhaps one bubble growing smoothly as the system expands. However, there is little distinction between spinodal decomposition and nucleation in a highly supercooled system [11].

To understand why these baryon density fluctuations can exceed our baseline estimate (1), suppose that by a proper time τ_Q the phase transition has created a relatively stable initial fraction $1-f$ of the low density bubbles within the high density phase. The net baryon density is then $\rho_B = f\rho_q + (1-f)\rho_h$, where ρ_q and ρ_h are the equilibrium densities of the respective phases. This density corresponds to a net rapidity density of baryons $N_B \approx \mathcal{A}\rho_B\tau_Q$, where $\mathcal{A}$ is the transverse area of the collision volume. We write

$$N_B \approx fN_q + (1-f)N_h, \tag{2}$$

where $N_{q,h} \equiv \rho_{q,h}\mathcal{A}\tau_Q$. The fluctuations of N_B are enhanced relative to a uniform system because the distribution of densities within each event is bimodal, with peaks at ρ_q and ρ_h. The variance is therefore

$$\mathcal{V}_B \approx \mathcal{V}_B^0 + f(1-f)(N_q - N_h)^2, \tag{3}$$

where $\mathcal{V}_B^0 = f\mathcal{V}_q + (1-f)\mathcal{V}_h$ is the weighted average of the variance of each phase. If we take each component to be a nearly ideal gas, then $\mathcal{V}_{q,h} = N_{q,h} + \overline{N}_{q,h}$, where N and $\overline{N}$ are the rapidity densities of baryons and antibaryons [4]. The quantity $\mathcal{V}_B^0$ then reduces to (1), precisely the Poissonian fluctuations we would expect if the system were uniform. The total variance (3) exceeds this value by an amount proportional to the square of the density contrast of the two phases. A nonequilibrium model of the quench would be needed to compute f. Observe that the two-phase effect, which vanishes with the density contrast, gives way to the critical divergence of (1) near tricritical point.

To estimate the affect of flow on baryon evolution, we write the net baryon current conservation law:

$$\partial\rho_B/\partial\tau + \partial_\mu j_B^\mu = 0. \tag{4}$$

where $\partial/\partial\tau \equiv u^\mu\partial_\mu$ for a fluid of four velocity u^μ. The flow of the system changes the baryon density through u^μ. If we neglect dissipation for the moment, then the net baryon number flows along with fluid as a whole, i.e. $j_B^\mu = \rho_B u^\mu$. For the Bjorken scaling flow, $u^\mu = u_{\rm s}^\mu \equiv \tau^{-1}(t,0,0,z)$ and $\tau \approx \sqrt{t^2 - z^2}$. As is well known, the density satisfies $\rho_B(\tau)\tau = \rho_B(\tau_Q)\tau_Q$ for scaling flow. The rapidity density is then τ independent. The variance of N_B for an ensemble of events i is $\mathcal{V}_B = \sum_i (N_B^i - N_B)^2$. Differentiating, we find the scaling results

$$dN_B/d\tau \approx 0 \quad \text{and} \quad d\mathcal{V}_B/d\tau \approx 0. \tag{5}$$

Even though the expansion after τ_Q will cause $f(\tau)$ to decrease, both N_B and $\mathcal{V}_B$ are fixed at the initial values (2, 3). It follows that (3) can represent the *observed* fluctuations, provided that the flow satisfies scaling.

Baryon diffusion can play an important role in both the onset and the propagation of fluctuations. We follow [13] and write $j_B^\mu = \rho_B u^\mu + j_{\rm diss}^\mu$, where we define u^μ so that the total momentum density of the fluid vanishes in the local rest frame. The diffusion of baryons relative to the fluid center of momentum gives rise to:

$$j_{\rm diss}^\mu = -MT\nabla^\mu\left(\mu_B/T\right), \tag{6}$$

where $M = D\partial\rho_B/\partial\mu_B$ is loosely termed the mobility, D is the diffusion coefficient and $\nabla^\mu = (g^{\mu\nu} + u^\mu u^\nu)\partial_\mu$.

To illustrate the onset of spinodal decomposition, we modify (6) to describe the strong inhomogeneities that spontaneously arise in the fluid. We follow the classic linear stability analysis of Cahn [12], duplicating the salient details here *a*) to motivate QCD calculations of the microscopic inputs and *b*) to highlight the differences and similarities with DCC studies [14]. We study the evolution of a small perturbation $\tilde\rho_B(k,t) = \tilde\rho_B(t)\exp(i\vec{k}\cdot\vec{r})$ in the spinodal region, where $\partial\rho_B/\partial\mu_B$ is negative. Anticipating that the fluid will spontaneously become inhomogeneous, we assume that the free energy of ref. [1] can be written as an effective Ginzburg-Landau functional of the baryon density,

$$F\{\rho_B(\vec{r})\} = \int d^3r\{f[\rho_B(\vec{r})] + \xi(\nabla\rho_B)^2/2\}, \tag{7}$$

in the local rest frame, where $f = \Omega/V + \mu_B\rho_B$ is the Helmholtz free energy density for a uniform system. Replacing $\mu_B = (\partial F/\partial N_B)_{T,V}$ in (6) with the functional derivative with respect to ρ_B, we find that

$$\frac{\partial}{\partial t}\tilde{\rho}_B(t) = -Mk^2\left(\frac{\partial \mu_B}{\partial \rho_B} + \frac{\xi}{M}k^2\right)\tilde{\rho}_B(t). \tag{8}$$

For $\partial\rho_B/\partial\mu_B < 0$, a perturbation of $k < k_{\rm c} = [|\partial\rho_B/\partial\mu_B|/M\xi]^{1/2}$ grows exponentially. The fastest mode at $k_{\rm sp} = k_{\rm c}/\sqrt{2}$ grows at the shortest time scale

$$\tau_{\rm sp} = \frac{2|\partial\rho_B/\partial\mu_B|}{Mk_{\rm sp}^2}, \quad \text{where} \quad k_{\rm sp} = \frac{k_{\rm c}}{\sqrt{2}}. \tag{9}$$

This mode dominates the early stage of the decomposition. Outside the spinodal region where $\partial\rho_B/\partial\mu_B > 0$, perturbations decay exponentially at rates that differ from normal diffusion only for wavelengths smaller than $2\pi/k_{\rm c}$ (roughly the equilibrium droplet size in a vacuum). At these early times, the variance averaged over the thermal fluctuations within an event grows as $\langle\langle\tilde{\rho}_B(r,t)^2\rangle\rangle \propto {\rm e}^{2t/\tau_{\rm sp}}$. This growth exceeds that of the density, $\propto {\rm e}^{t/\tau_{\rm sp}}$, allowing large fluctuations to build.

A quantitative treatment of spinodal decomposition using modern techniques [7] awaits the introduction of kinetic terms to the coarse-grained free energy density [1,2] — neither M nor ξ are known. However, we can get a rough idea of the time scales needed for the onset of instabilities by identifying the inverse momentum $k_{\rm sp}$ with the correlation length, $m_\sigma^{-1} \sim 1$ fm, as in [3]. Well within the spinodal region, we surmise that $|M(\partial\rho_B/\partial\mu_B)^{-1}|$ is of the order of the diffusion coefficient in a stable, perturbative plasma, $\sim 1-3$ fm [9]. Therefore $\tau_{\rm sp} \sim 1$ fm or smaller, suggesting that spinodal decomposition can erupt violently when a quench is achieved.

One faces a similar situation in disoriented chiral condensate formation [14]. There, one studies nonequilibrium fluctuations of the chiral order parameter at very small baryon density. Equilibrium fluctuations of that order parameter diverge along with the baryon density fluctuations at the tricritical point [1]. The essential distinction between our first order transition and those phenomena is that, here, the low density phase can coexist with the high density phase in thermodynamically stable bubbles. This is impossible in a second order transition. The wavelength scale $\sim 2\pi/k_{\rm sp}$ of the fastest growing mode (9) is a nonequilibrium manifestation of the equilibrium size of bubbles. The second order chiral transition is also heralded by unstable modes, but the fastest mode has $k = 0$, reflecting the critical divergence of the correlation length. Furthermore, the corresponding time scale diverges in a "critical slowing down" as the correlation length $\sim m_\sigma^{-1} \to \infty$, see, e.g., eq. (2) in [15]. Analogous behavior takes place at the tricritical point, where $M^{-1}\partial\rho_B/\partial\mu_B$ and (9) diverge.

Optimistic that fluctuations can be produced by a quench, we turn to the dissipation of these fluctuations due to diffusion. For a stable liquid, we use (4,6) to obtain the diffusion equation,

$$\partial\rho_B/\partial\tau + \rho_B\partial_\mu u^\mu = D\nabla^2\rho_B, \tag{10}$$

where we take D to be constant and neglect thermodiffusion. Observe that diffusion *cannot* affect scaling flow because the diffusive term vanishes if ρ_B is a function of τ alone. In that case (5) would still apply. Let us then face a conservative scenario in which phase separation only occurs as a perturbation $\tilde{\rho}(\tau,\eta)$ of the scaling hadronic system (here $\tanh\eta = z/t$ is the spatial rapidity). To be concrete, we take N_B and $\mathcal{V}_B$ to be perturbed at time τ_Q by $\tilde{N}_B \sim f(N_q - N_h)$ and $\tilde{\mathcal{V}}_B \sim f(1-f)(N_q - N_h)^2$ for $f = f_0\exp\{-\eta^2/2\sigma^2\}$. Note that $\sigma \approx 0.88$ for a spherically symmetric perturbation.

We describe the evolution for $\tau > \tau_Q$ using (10) for $\rho_B\partial_\mu u^\mu \approx \rho_B/\tau$. Writing the rapidity density at fixed τ as $N_B(\tau,\eta) = \rho_B(\tau,\eta)\mathcal{A}\tau$, we obtain

$$\tau^2\partial\tilde{N}_B/\partial\tau = D\partial^2\tilde{N}_B/\partial\eta^2. \tag{11}$$

The unperturbed rapidity density N_B^0 is constant. It follows that $\tilde{N}_B = \tilde{N}_B(\tau_Q)\phi_\sigma(\eta)$, where $\tilde{N}_B(\tau_Q) = f_0(N_q - N_h)$ and

$$\phi_\sigma = \frac{\exp\{-\eta^2/2(\sigma^2 + 2Ds)\}}{(1 + 2Ds\sigma^{-2})^{1/2}}, \qquad s = \tau_Q^{-1} - \tau_F^{-1}, \tag{12}$$

and τ_F is the freezeout time. Longitudinal flow limits the degree to which the Gaussian perturbation can be dispersed. For $\tau_F \gg \tau_Q$, we see that the rapidity density near $y \approx \eta \approx 0$ is $N_B \approx N_B^0 + \tilde{N}_B(\tau_Q)\{1 + 2D/(\tau_Q\sigma^2)\}^{-1/2}$ since $s \approx \tau_Q^{-1}$.

To study the contribution of the perturbation to the variance for $\tau \gg \tau_Q$, we write the event averaged $\mathcal{V}_B \approx \mathcal{V}_B^0 + 2\langle N_B^0\tilde{N}_B\rangle$ [16]. We then differentiate $\mathcal{V}_B$ with respect to τ and use (11) to obtain:

$$\tau^2\partial\tilde{\mathcal{V}}_B/\partial\tau = D\partial^2\tilde{\mathcal{V}}_B/\partial\eta^2, \tag{13}$$

to linear order in $\tilde{N}_B/N_B$. We find $\tilde{\mathcal{V}}_B = (N_q - N_h)^2\{f_0\phi_\sigma(\eta) - f_0^2\phi_{\sigma/\sqrt{2}}(\eta)\}$, for $\phi_\sigma(\eta)$ given by (12).

The diffusion coefficient for baryon current is unknown for the mixed phase described in [1,2]. However, we can get a rough estimate for D in the pure phases from kinetic theory, which implies that $D \sim \tau_{\rm diff}\, v_{\rm th}^2/3$, where $v_{\rm th}$ is the thermal velocity of baryons and $\tau_{\rm diff}$ is the relaxation time for diffusion. For nucleons diffusing through a hadron gas, $\tau_{\rm diff} \sim 35$ fm and $D \sim 6$ fm [8] at a temperature of 150 MeV. Flavor diffusion through a perturbative quark gluon plasma yields $D \sim 1-3$ fm [9]. We estimate diffusion in the mixed phase using the larger hadronic value, $D \sim 6$ fm. In fact, arguments in ref. [1] suggest similarities between hadrons and droplets of mixed phase. At $\eta \approx y \approx 0$, we find the rather small decrease $\tilde{\mathcal{V}}_B/\tilde{\mathcal{V}}_B(\tau_Q) \sim 76\%$ for $\tau_Q \approx 5$ fm, $\sigma = 0.88$, $\tau_F \to \infty$ and $f_0 = 1/2$.

Experimenters can use baryon fluctuations to search for the tricritical point as follows. Collisions for a range of beam energies, ion combinations and centralities can produce high density systems that follow trajectories as in fig. 1. Results can be compared as shown in fig. 2 by plotting the normalized ratio [17],

$$\omega_B \equiv \mathcal{V}_B/(N+\overline{N}) \tag{14}$$

as a function of a normalized centrality selector, e.g. the total charged particle multiplicity $N_{\rm ch}$. In the absence of unusual fluctuations, this ratio is energy independent, with a value close to unity, cf. eq. (1). In fig. 2 we show the results of simulated collisions incorporating the above results to compute the mean and variance in the event generator of refs. [4,5]; see these refs. for details. The rapidity densities of baryons, antibaryons and charged hadrons are taken to be 60, 15 and 300 for impact parameter $b=0$ and scale with the number of participants for $b>0$, as appropriate at SPS energy. A value of f is assigned to each event using the ad hoc distribution, $f(b) = 0.25\ [1-(b/b_0)^2]$ for $b<b_0=3$ fm. The mean baryon number and its fluctuations are computed at $y\approx\eta=0$ for $\tau_Q=5$ fm, $\tau_F=10$ fm, $D=6$ fm, and rapidity density contrasts $\delta N = N_q - N_h = 20$ and 40, corresponding to $\rho_q-\rho_h\sim 0.10$ and 0.2 fm^{-3} on the scale of normal nuclear matter density. The 'hadron' curve is computed assuming no enhancement. The difference between this curve and unity is due to impact parameter fluctuations (volume and thermal fluctuations [4,5] are omitted). The top curve is computed with $\delta N=40$ but without diffusion. We see that diffusion is a small effect for our parameter choices.

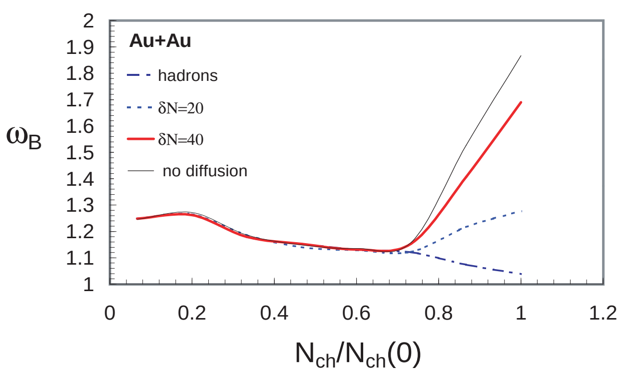

FIG. 2. Variance (14) as a function of charged particle multiplicity normalized to the average value at $b=0$.

Much more work is needed to obtain crucial information, such as the beam energies and rapidity ranges needed to probe the high density phase. To start, we need a better idea of the phase diagram away from the tricritical point. We must also understand transport processes in the unstable and mixed phase regions. Then, we can combine these ingredients to perform dynamical simulations such as those in [7] to assess the likelihood of a quench. We have also overlooked the possibility of supeconductivity in the high density phase, a reasonable assumption near the tricritical point [1]. Superconductivity would modify baryon transport because a) the condensate carries baryon current and b) additional transport modes are possible.

In summary, we have explored the possibility that the phase transition of refs. [1,2] can enhance fluctuations of the net baryon number. We find that fluctuations can plausibly persist through freezeout to surmount impact-parameter fluctuations [17]. A collision that reaches the tricritical point will have the largest fluctuations, but collisions that pass below can also be extraordinary, provided that the dynamics can quench the system. Calculations for $y\approx 0$ at RHIC energy [5] suggest that proton fluctuations alone can reveal net baryon fluctuations at the level of fig. 2. Work is in progress to evaluate this signal at higher rapidity and lower beam energy where higher baryon densities are met.

I am grateful to C. Pruneau for many illuminating discussions, and to J. Berges, K. Rajagopal, E. Surdutovich and S. Voloshin for useful comments. I also thank the Aspen Center for Physics for hospitality during the completion of this work. This work is supported in part by the U.S. DOE grant DE-FG02-92ER40713.

---

[1] J. Berges and K. Rajagopal, Nucl. Phys. B538 215 (1999), hep-ph/9804233.
[2] M.A. Halasz, A.D. Jackson, R.E. Schrock, M.A. Stephanov and J.J.M. Verbaarschot, Phys. Rev. D 58, 096007 (1998), hep-ph/9804290.
[3] M. Stephanov, K. Rajagopal and E. Shuryak, Phys Rev Lett 81, 4816 (1998); hep-ph/9903292.
[4] S. Gavin and C. Pruneau, nucl-th/9906060.
[5] S. Gavin and C. Pruneau, nucl-th/9907040.
[6] H. Heiselberg and A.D. Jackson, nucl-th/9809013; Z. Huang, I. Sarcevic, R. Thews, Xin-Nian Wang Phys. Rev. D54:750-758, (1996), hep-ph/9511387.
[7] G. Brown, P. A. Rikvold, M. Sutton, M. Grant, cond-mat/9905343 and refs. therein; F. Abraham, Phys. Rep. 53, 93 (1979).
[8] M. Prakash, M. Prakash, R. Venugopalan and G. Welke Phys. Rept. 227, 321, (1993).
[9] H. Heiselberg, C.J. Pethick Phys. Rev. D48, 2916, (1993).
[10] P. Bozek, Y.B. He, J. Hüfner, nucl-th/9806066 and refs. therein; P. Chomaz, M. Colonna, A. Guarnera, J. Randrup, Phys. Rev. Lett. 73, 3512, (1994).
[11] I. Mishustin, Phys. Rev. Lett., 82, 4779 (1999), nucl-th/9809013.
[12] J.W. Cahn, Acta Metall. 9, 795 (1961); J. Chem. Phys. 42, 93, (1965).
[13] L.D. Landau and E. M. Lifshitz, Fluid Mechanics, (Pergamon, 1959), sec. 127.
[14] K. Rajagopal, Quark Gluon Plasma, vol.2, R.C. Hwa, ed., (World Scientific, 1995) pp 484-554.
[15] S. Gavin, A. Gocksch and R.D. Pisarski, Phys. Rev. Lett. 72, 2143, (1994).
[16] Linear contributions to the variance vanish during the onset of spinodal decomposition on averaging over fluctuations. At later times, fluctuations are less important, so that this term can contribute.
[17] G. Baym and H. Heiselberg, nucl-th/9905022.